

Opinions: Online Appendix

You may wonder why scholars often find the S-shaped curve of diffusion within the population. Why does it follow from the social learning theory? At his time, Tarde did not use formal models to express his intuition in a more precise way. It is at this point, that researchers have used such formal models as a tool to better understand the diffusion of innovations. In the literature, many models have been proposed (Valente, 2010). Importantly, however, it appears that even very simple formal models of social learning do well in predicting this S-shape curve.

Let's have a look at such a *simple diffusion model* (Page, 2018; Valente, 2010). In this model, we assume that we have a population N of fixed size. Then, there are two groups at any time t : those who have adopted the innovation, and those who have not. Hence, $\frac{At}{N}$ indicates the proportion of people in the population who have adopted the innovation at t . The proportion that has not adopted the innovation is then the rest of the population: $\frac{N-At}{N}$. We assume that people meet in pairs, and therefore the probability that a random meeting is a pair of an adopter and a non-adopter is the multiplication of the two proportions: $\frac{At}{N} \cdot \frac{(N-At)}{N}$. It is furthermore assumed that once someone has adopted the innovation, he will remain so.

Following social learning theory, we assume that people adopt the innovation, when they come into contact with those who have already adopted, because they learn from others. However, it seems reasonable to assume that not every contact with a person who has already adopted will directly result in the adoption of the innovation. That is to say, social influence will not occur always, for various reasons. The non-adopter might be skeptical about the innovation, he might not dare to take the risk, or maybe he did not receive sufficient information about the innovation from just one chat with the adopter. We thus introduce another variable, s , which reflects this degree of social influence between pairs of adopters and non-adopters. We then get the following:

$$s \frac{At}{N} \frac{(N-At)}{N}$$

This model gives us the probability that a random interaction between a pair of individuals results in an adoption of the innovation. But how many interactions are there? How often do adopters communicate about their innovation to others? To indicate this frequency of contacts for each individual in the population, the model introduces the variable c . The total number of interactions is then a multiplication of c with the population size, N . The simple diffusion model we then get is:

$$A_{(t+1)} = A_t + Ncs \frac{At}{N} \frac{(N-At)}{N}$$

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Essentially, this model says that the total number of people who have adopted at a certain time A_{t+1} is the same as the number who adopted the time before A_t , plus the new-adopters:
$$Nc s \frac{A_t}{N} \frac{(N-A_t)}{N}.$$

Do we get the S-shaped curve with this model? Indeed, this is what the model predicts. To illustrate this with some fictitious empirical data, consider Table 5A. It consists of a population N of 100 persons. The social influence rate s is 0.33, which means that the probability that a non-adopter adopts the innovation when meeting someone else who already adopted is about one third. Further, we assume that each individual has 3 social interactions within each time period, hence $c = 3$. What happens?

Table 5A Illustration of simple diffusion model with fictitious data

Time (t)	Adopters (A)	Population (N)	Social influence (s)	Contacts (c)	%Adopters (At/N)	%non-adopters ((N-At)/N)	New adopters
0	0	100	0.33	3	0	1	0
1	1	100	0.33	3	0.01	0.99	0.98
2	1.98	100	0.33	3	0.02	0.98	1.92
3	3.90	100	0.33	3	0.04	0.96	3.71
4	7.61	100	0.33	3	0.08	0.92	6.96
5	14.58	100	0.33	3	0.15	0.85	12.33
6	26.90	100	0.33	3	0.27	0.73	19.47
7	46.37	100	0.33	3	0.46	0.54	24.62
8	70.99	100	0.33	3	0.71	0.29	20.39
9	91.38	100	0.33	3	0.91	0.09	7.80
10	99.18	100	0.33	3	0.99	0.01	0.81
11	99.99	100	0.33	3	1.00	0.00	0.01

At t_0 nobody in the population has adopted the innovation yet. That means that the number of adopters, A , equals 0. Suppose that at t_1 one person –an innovator– adopts the innovation. The model then predicts that this person (1% of the population), interacts with the remaining 99% who have not yet adopted the innovation, and that this person transmits the innovation to them. He has 3 interactions, and of these three interactions each interaction has a probability of 0.33 to result in an adoption. This means that at t_1 , the 1 innovator, diffuses the innovation to around 1 person. We can see this when we fill in the equation for the new adopters:

$$100 * 3 * 0.33 \frac{1}{100} \frac{(100 - 1)}{100} = 0.98$$

This means that at t_2 the population consists of 1.98 adopters. These adopters interact with the remaining 98% of the population of non-adopters. This results in 1.92 new adopters. It

takes some time for the diffusion to accelerate, which is highest at t_7 , when the number of new adopters is 24.62. As you can see from this example, the cumulative number of people who adopted follows an S-shaped curve, and the number of new adopters resembles a normal distribution.

From this model, we can learn something about diffusion dynamics, and why this has an S-shape curve. In the beginning period, there are very few new adopters, because there are very few people from which they could learn about the innovation. Thus, when $\frac{At}{N}$ is low, the other group, $\frac{N-At}{N}$, cannot learn about the innovation, because they rarely come into contact with the adopters. At the end period, there are again very few new people who adopt, but here the reason is that $\frac{N-At}{N}$ is low, and hence the majority of adopters tend to interact with others adopters, and hardly ever meet non-adopters. The rise in adoption peaks in between these extremes, when groups are similar in size. This explains why the diffusion of innovations follows an S-shaped curve.

From this insight, it follows that the S-shape diffusion curve is independent from the size of the population N , the contact frequency c , and the social influence rate s . In both large and small populations, the diffusion process will follow this S-shape according to the model. Likewise, it will do so for groups of people who have many or few contacts, and for low and high transmission rates. To see this, consider Figure 5A, which presents the diffusion process of Table 5A (and in which $s=0.33$), together with another one, which has a lower social influence rate, namely 0.2. In reality, this could reflect the difference between an innovation that is high attractive and hence easily transmitted from person to person ($s = 0.33$) and another innovation which is for whatever reason less attractive ($s = 0.20$). The figure shows that both innovations will eventually be adopted by the entire population, following the predicted S-shape curve. The only difference is the speed of innovation: the highly attractive innovation is adopted faster than the less-attractive one.

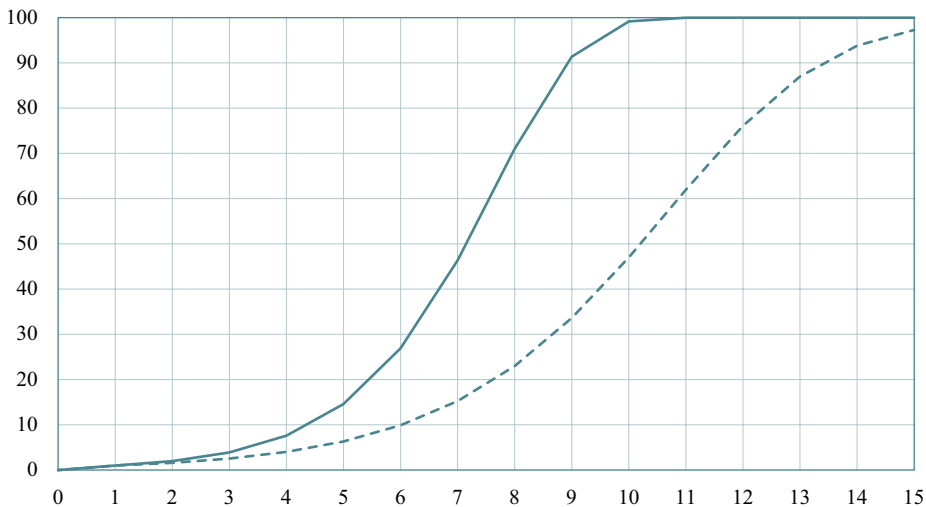


Figure 5A Two diffusion curves, with different social influence rates ($s = 0.33$ and $s = 0.2$)

References

Page, S. E. (2018). *The Model Thinker*. New York, NY: Basic Books.

Valente, T. W. (2010). *Social Networks and Health: Models, Methods, and Applications*. New York, NY: Oxford University Press.